## Contextual Representation and Reasoning with Description Logics

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## Motivation

- Information on the SW is valid w.r.t. some assumed context


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## MATCHES

Presented by
Emirates

| Group stage | Stage 2 | Calendar |
| :---: | :---: | :---: |



Cape Town－Green Point Stadium

## Summary



Italy and Paraguay share
spoils
Italy opened their FIFA World Cup ${ }^{\text {m }}$ defence with a come－from－behind 1－1 draw in Group $F$ against Paraguay on a rainy Monday night in Cape Town．

FIFA．com＇s Focus
目 Italy meet their match
回 Honda creates history for Japan
目 Danes no match for Dutch
，＊nominis nornoct
Fombuesere Man of the Match

## ED HYunaal Fan Photo



The Fan of the Tournament vote is now closed．
Ched back soon to
find out which fan
is the lucky winner
of a brand new
of a brand new
Hyundai i10！
More＂


| Statistics |  |  |
| :--- | :---: | ---: |
| $\mathbf{4}$ Italy |  | Paraguay |
| $\mathbf{1 0}$ | Shots | $\mathbf{8}$ |
| $\mathbf{8}$ | Comer kicks | $\mathbf{4}$ |
| $\mathbf{1}$ | Yellow cards |  |
| $\mathbf{0}$ | Second yellow <br> card and red <br> card | $\mathbf{1}$ |
| $\mathbf{0}$ | Red Cards | $\mathbf{0}$ |
| $\mathbf{5 2 \%}$ | Possession（\％） | $\mathbf{0} \%$ |

## Logic of Context (McCarthy 1993)

- If context of formulae $\phi$ and $\psi$ is different: introduce context identifiers $c_{1}, c_{2}, \ldots$ and special predicate ist/2
use $\operatorname{ist}\left(c_{1}, \phi\right)$ and $\operatorname{ist}\left(c_{2}, \psi\right)$


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- knowledge lifting:
$\forall x$ ist $\left(c_{1}\right.$, Winner $\left.(x)\right) \rightarrow \operatorname{ist}\left(c_{2}\right.$, Team $\left.(x)\right)$


## Context as a Box (Benerecetti et al. 2000)



## Dimensional Space (CYC, Lenat 1998)



- Context can be organized in dimensional space
- W.r.t. narrower-broader relation


## CKR: Objectives

- Tailor the logic of context for SW
- Reasoning tasks: subsumption, entailment, query answering
- Develop a tractable version
- Implement a working prototype
- Evaluate rep. power \& performance


## Preliminaries: Description Logics

- Language
- Individuals $a, b, \ldots$
- Concepts $A, B, C \ldots$
- Roles $R, S, \ldots$
- Complex concepts
- $C::=A|\neg C| C \sqcap D|C \sqcup D| \exists R . C \mid \forall R . C$
- $\mathrm{KB} \mathcal{K}=\langle\mathcal{T}, \mathcal{A}\rangle$ contains axioms:
- Subsumption $C \sqsubseteq D$
- Class assertions $C(a)$
- Role assertions $R(a, b)$


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## Preliminaries: Semantics

Interpretation $\mathcal{I}=\left\langle\Delta^{\mathcal{I}},,^{\mathcal{I}}\right\rangle, \Delta^{\mathcal{I}} \neq \emptyset$

| $\neg C$ | $\Delta^{\mathcal{I}} \backslash C^{\mathcal{I}}$ |
| :--- | :--- |
| $C \sqcap D$ | $C^{\mathcal{I}} \cap D^{\mathcal{I}}$ |
| $C \sqcup D$ | $C^{\mathcal{I}} \cup D^{\mathcal{I}}$ |
| $\exists R . C$ | $\left\{x \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}}\langle x, y\rangle \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\right\}$ |
| $\forall R . C$ | $\left\{x \in \Delta^{\mathcal{I}} \mid \forall y \in \Delta^{\mathcal{I}}\langle x, y\rangle \in R^{\mathcal{I}} \Longrightarrow y \in C^{\mathcal{I}}\right\}$ |
| $C \sqsubseteq D$ | $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ |
| $C(a)$ | $a^{\mathcal{I}} \in C^{\mathcal{I}}$ |
| $R(a, b)$ | $\left\langle a^{\mathcal{I}}, b^{\mathcal{I}}\right\rangle \in R^{\mathcal{I}}$ |

- $\mathcal{I}$ is a model of $\mathcal{K}$ iff $\mathcal{I} \models \phi$ for all $\phi \in \mathcal{T} \cup \mathcal{A}$
- $\mathcal{K} \vDash C$ iff $C^{\mathcal{I}} \neq \emptyset$ in some model of $\mathcal{K}$
- $\mathcal{K} \models C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ in all models of $\mathcal{K}$


## Syntax: Contexts

world, sports, anytime

world, footbal, 2010

world, ice_hockey, 2010

world, FIFA_WC, 2010


world, IHWC, 2010

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world, ice_hockey, 2010

world, IHWC, 2010
Italy, NFL, 2010
$\mathcal{S R O I Q}$ $\square$

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world, IHWC, 2010


## Syntax: Meta-knowledge



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- context IDs




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- context IDs
- dimensional values location $\left(\mathcal{C}_{1}\right.$, Italy $)$

location:=Italy


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- dimensional values location ( $\mathcal{C}_{1}$, Italy) topic ( $\left.\mathcal{C}_{1}, \mathrm{NFL}\right)$ time $\left(\mathcal{C}_{1}, 2010\right)$


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Italy, NFL, 2010


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Italy $\prec$ World FIFA_WC $\prec$ football football $\prec$ sports

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## Hierarchy of Contexts



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- new symbol $X_{A_{i_{1}}:=d_{1}, \ldots, A_{i_{n}}:=d_{n}}$


## Syntax: Inside Contexts

- Locality of knowledge
- Context/roles $X$ have independent meaning in different contexts
- Qualified symbols: refer to entities from different context
- new symbol $X_{A_{i_{1}}:=d_{1}, \ldots, A_{i_{n}}:=d_{n}}$
- for all concept/role $X$
- for all dimensional vectors $A_{i_{1}}:=d_{1}, \ldots, A_{i_{n}}:=d_{n}$


## Syntax: Qualified Symbols

- Example usage:
world, FIFA_WC, 2010
Team $\sqsubseteq$ NationalTeam world,football, $2010^{\text {M MenTeam }}$ world,football, 2010 Team(Team_Italy)
Team(Team_Paraguay)
Referee(Benito_Archundia) has_coach(Team_Italy, Marcello_Lippi)


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    Team(Team_Italy)
    Team(Team_Paraguay)
    Referee(Benito_Archundia)
    has_coach(Team_Italy, Marcello_Lippi)
    plays_for(Daniele_Derossi, Team_Italy)
    plays_for ltaly,NFL
plays_for(Nelson_Valdez,Team_Paraguay)
plays_forGermany,NFL
```


## CKR: Semantics

$$
\mathfrak{I}=\left\{\mathcal{I}_{\mathbf{d}}\right\}_{\mathcal{C}_{\mathbf{d}} \in \mathfrak{K}} \quad \Delta_{\mathbf{d}} \subseteq \Delta_{\mathbf{e}} \text { if } \mathbf{d} \prec \mathbf{e}
$$

1. $\left(\top_{\mathbf{d}}\right)^{\mathcal{I}_{f}} \subseteq\left(\top_{e}\right)^{\mathcal{I}_{f}}$ if $\mathbf{d} \prec \mathbf{e}$
2. $\left(A_{\mathbf{f}}\right)^{\mathcal{I}_{\mathrm{d}}} \subseteq\left(T_{\mathbf{f}}\right)^{\mathcal{I}_{\mathrm{d}}}$
3. $\left(R_{\mathbf{f}}\right)^{\mathcal{I}_{\mathbf{d}}} \subseteq\left(\top_{\mathbf{f}}\right)^{\mathcal{I}_{\mathbf{d}}} \times\left(\top_{\mathbf{f}}\right)^{\mathcal{I}_{\mathbf{d}}}$
4. $a^{\mathcal{I}_{\mathrm{d}}}=a^{\mathcal{I}_{\mathrm{e}}}$, given $\mathbf{d} \prec \mathbf{e}$, either if $a^{\mathcal{I}_{\mathrm{d}}}$ is defined, or if $a^{\mathcal{I}_{e}}$ is defined and $a^{\mathcal{I}_{e}} \in \Delta_{\mathrm{d}}$
5. $\left(X_{\mathbf{d}_{\mathrm{B}}}\right)^{\mathcal{I}_{\mathrm{e}}}=\left(X_{\mathrm{d}_{\mathrm{B}}+\mathrm{e}}\right)^{\mathcal{I}_{\mathrm{e}}}$
6. $\left(X_{\mathbf{d}}\right)^{\mathcal{I}_{\mathrm{e}}}=\left(X_{\mathbf{d}}\right)^{\mathcal{I}_{\mathrm{d}}}$ if $\mathbf{d} \prec \mathbf{e}$
7. $\left(A_{\mathbf{f}}\right)^{\mathcal{I}_{\mathbf{d}}}=\left(A_{\mathbf{f}}\right)^{\mathcal{I}_{\mathbf{e}}} \cap \Delta_{\mathbf{d}}$ if $\mathbf{d} \prec \mathbf{e}$
8. $\left(R_{\mathbf{f}}\right)^{\mathcal{I}_{\mathrm{d}}}=\left(R_{\mathbf{f}}\right)^{\mathcal{I}_{\mathrm{e}}} \cap\left(\Delta_{\mathbf{d}} \times \Delta_{\mathbf{d}}\right)$ if $\mathbf{d} \prec \mathbf{e}$
9. $\mathcal{I}_{\mathbf{d}}=\mathrm{K}\left(\mathcal{C}_{\mathbf{d}}\right)$

## Characterization: Embedding @d



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- $(C \sqsubseteq D) @ \mathbf{d}=C @ \mathbf{d} \sqsubseteq D @ \mathbf{d}$


## Characterization: Embedding @d



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Properties: Knowledge Propagation over Common Super-context


## Properties: Knowledge Propagation over Common

## Super-context



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## Reasoning in CKR

- DL-based CKR ................................. tableaux algorithm
- OWL RL-base CKR .................. tractable forward chaining


## $\mathcal{A L C}$ Tableau Algorithm

$$
\forall R . \neg C \sqcap(\exists R . C \sqcup \exists R . D)
$$

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## $\mathcal{A L C}$ Tableau Algorithm

$\forall R . \neg C \sqcap(\exists R . C \sqcup \exists R . D)$
$\forall R . \neg C, \exists R . C \sqcup \exists R . D$

## $\mathcal{A L C}$ Tableau Algorithm

$$
\begin{aligned}
& \forall R . \neg C \sqcap(\exists R . C \sqcup \exists R . D) \\
& \forall R . \neg C, \exists R . C \sqcup \exists R . D \\
& \exists R . C \\
& \text { • }
\end{aligned}
$$

## $\mathcal{A L C}$ Tableau Algorithm



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## $\mathcal{A L C}$-based CKR Tableau Algorithm



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## $\mathcal{A L C}$-based CKR Tableau Algorithm



## OWL 2 RL-based CKR Tableau Algorithm

- Forward chaining algorithm
- Extends OWL RL algorithm by a number of rules

$$
\begin{aligned}
& \mathbf{h}:\left(\operatorname{ardf}: \operatorname{type} C_{\mathbf{d}}\right) \\
& \mathbf{g} \preceq \mathbf{h} \\
& \text { presentIn }(\mathbf{g}, a)
\end{aligned} \quad \Longrightarrow \quad \mathbf{g}:\left(a \operatorname{rdf}: \operatorname{type} C_{\mathbf{d}}\right)
$$

## Implementation

- Implementation for OWL 2 RL fragment
- Implemented over Sesame RDF data store
- Contexts implemented as named graphs
- Propagation rules implement CKR semantics
- Contextualized queries


## Use Case

- How can one model with this thing?
- How does it perform?


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- FIFA World Cup domain
- Suitable for contextualization
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- How can one model with this thing?
- How does it perform?
- FIFA World Cup domain
- Suitable for contextualization
- One edition $\sim 50$ contexts
- Compare w flat RDF model
- Evaluate query execution times
- See:
https://dkm.fbk.eu/images/4/4a/TR-FBK-DKM-2011-3.pdf


## Contextualized Queries

- Contextualized extension of SPARQL
- See:
https://dkm.fbk.eu/images/2/2a/TR-FBK-DKM-2011-2.pdf


## Summary: Reasoning \& Complexity

|  | OWL 2 RL | $\mathcal{A L C}$ | $\mathcal{S R O I Q}$ |
| :--- | :---: | :---: | :---: |
| Reasoning | forward | tableaux | reduction |
| Complexity | PTime | ExPTiME | 2NEXPTiME |
| Implementation | Yes | - | - |
| Querying | Yes | - | - |

## Conclusion

- Contextualization is a demanding task for SW
- CKR offers a viable solution
- Supports OWL 2 or any of its fragment
- Tableau and tractable reasoning
- Invariant complexity
- Implementation
- Contextualized queries

