Towards Typed Higher-Order Description Logics

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Outline

Motivation

Typed Higher-Order DLs

Semantics

(Un)wanted Properties

Conclusions

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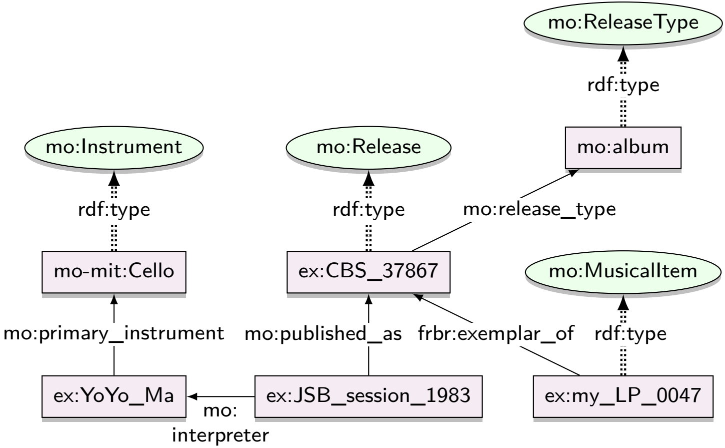
Motivation

PURO background modelling language

- capture ontological distinctions in foreground models
- particular–universal distinction
- relationship–object distinction
- intended to use with LD vocabularies

Svátek et al. (OWLED 2013, K-CAP 2013)

Motivation: Example – Music Ontology



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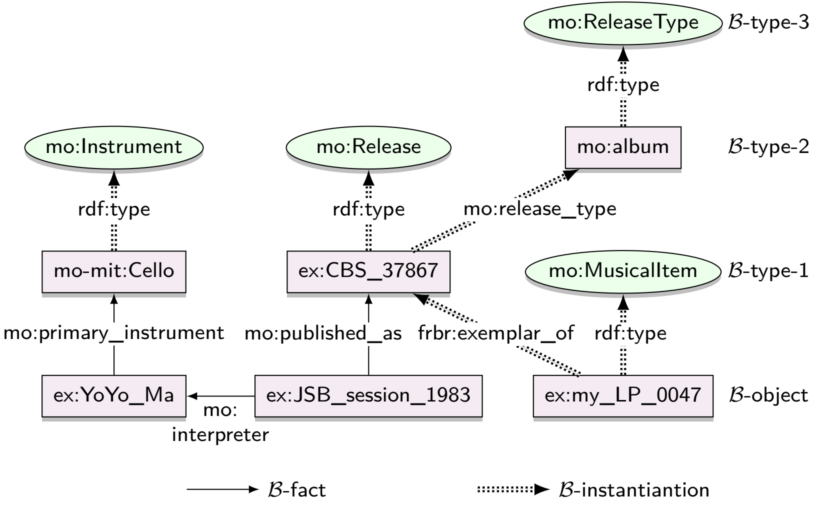

Motivation: Desiderata

To capture PURO background models in a DL-like language and reason with them:

- Higher-order classes – $\mathcal{B}$-types
- Roles between entities of different orders – $\mathcal{B}$-relations
- Homogeneity of
  - classes
  - role domains and ranges
- Suitable semantics
- ~~n-ary roles~~

Motivation: Existing Approaches

- Motik (2007)
- De Giacomo et al. (2009)

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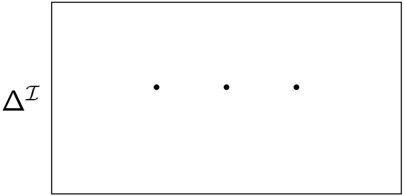

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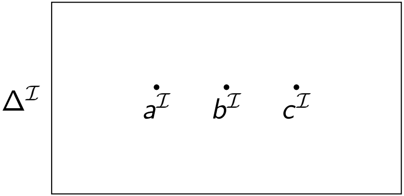

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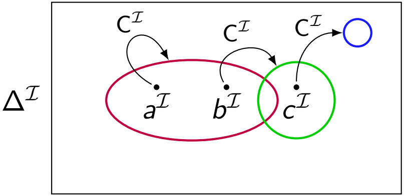

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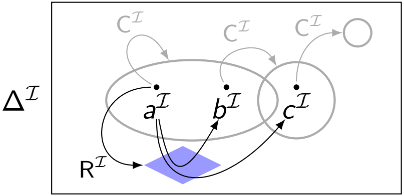

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Typed DL Vocabulary

Typed DL vocabulary is a disjoint union of a countable number of countable sets:

- N_C^t, for $t \geq 0$, the set of *concept names of type t* ($N_I = N_C^0$, the set of individual names)
- N_R^{tu}, for $t, u > 0$, the set of *role names between types t and u*

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Notation:

- $A^t, B^t, \ldots \in N_C^t$
- $R^{tu}, S^{tu}, \ldots \in N_R^{tu}$

Role Expressions in $\mathcal{TH}(\mathcal{SROIQ})$

The set of tu-role expressions of $\mathcal{TH}(\mathcal{SROIQ})$ is recursively defined as the smallest set containing:

- R^{tu}
- R^{ut-}
- U^{tu}
- $S_1^{t_1 u_1} \cdot S_2^{t_2 u_2} \cdot \dots \cdot S_n^{t_n u_n}$, s.t. $t_1 = t$, $u_n = u$, $u_i = t_{i+1}$ for all i

given atomic role R^{tu}, tu- and $t_i u_i$-role expressions S^{tu}, $S_i^{t_i u_i}$, and t, u, t_i, $u_i \geq 0$

Concept Descriptions in $\mathcal{TH}(\mathcal{SROIQ})$

The set of t-descriptions of $\mathcal{TH}(\mathcal{SROIQ})$ is recursively defined as the smallest set containing:

- A^t
- $\neg C^t$
- $C^t \sqcap D^t$
- $\exists R^{tu}.C^u$
- $\geqslant n\, R^{tu}.C^u$
- $\exists R^{tt}.\mathrm{Self}$
- $\{A^{t-1}\}$

given atomic concepts A^t and A^{t-1}, t- and u-descriptions C^t, D^t, C^u, tu- and tt-role expressions R^{tu}, R^{tt}, and $t, u > 0$

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Notation: $\top^t = A^t \sqcup \neg A^t$ for $t > 0$ and some $A^t \in N_C^t$.

Knowledge Bases in $\mathcal{TH}(\mathcal{SROIQ})$

$\mathcal{TH}(\mathcal{SROIQ})$ knowledge base $\mathcal{K}$ is a finite set of axioms of the following forms:

- $C^t \sqsubseteq D^t$
- $R^{tu} \sqsubseteq S^{tu}$
- $\mathrm{Ref}(R^{tu})$
- $\mathrm{Dis}(R^{tu}, S^{tu})$
- $A^{t-1} : C^t$
- $A^{t-1}, B^{u-1} : R^{tu}$
- $A^{t-1}, B^{u-1} : \neg R^{tu}$

given atomic concepts A^{t-1}, B^{u-1}, t-descriptions C^t, D^t, tu-role expressions R^{tu}, S^{tu}, and $t, u > 0$

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HiLog-style Interpretations

HiLog-style interpretation is a triple $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}, \cdot^{\mathcal{E}})$ s.t.:

- $\Delta^{\mathcal{I}} = \biguplus_{t \geq 0} \Delta_t^{\mathcal{I}} \uplus \biguplus_{t,u>0} \Delta_{tu}^{\mathcal{I}}$ and $\Delta_0^{\mathcal{I}} \neq \emptyset$,
- $A^{t\,\mathcal{I}} \in \Delta_t^{\mathcal{I}}$, for $A^t \in N_{\mathsf{C}}^t$ and $t \geq 0$
- $R^{tu\,\mathcal{I}} \in \Delta_{tu}^{\mathcal{I}}$, for $R^{tu} \in N_{\mathsf{R}}^{tu}$ and $t, u > 0$
- $c^{\mathcal{E}} \subseteq \Delta_{t-1}^{\mathcal{I}}$, for $c \in \Delta_t^{\mathcal{I}}$ and $t > 0$
- $r^{\mathcal{E}} \subseteq \Delta_{t-1}^{\mathcal{I}} \times \Delta_{u-1}^{\mathcal{I}}$, for $r \in \Delta_{tu}^{\mathcal{I}}$ and $t, u > 0$

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Notation:

- $X^{\mathcal{E}} := X^{\mathcal{I}\mathcal{E}} = (X^{\mathcal{I}})^{\mathcal{E}}$ for atomic concepts and roles

HiLog-style Interpretations (cont.)

$\boldsymbol{X}$	$\boldsymbol{X}^{\mathcal{E}}$
$\neg C^t$	$\Delta^{\mathcal{I}}_{t-1} \setminus C^{t\,\mathcal{E}}$
$C^t \sqcap D^t$	$C^{t\,\mathcal{E}} \cap D^{t\,\mathcal{E}}$
$\exists R^{tu}.C^u$	$\{\, x \mid \exists y.\langle x, y\rangle \in R^{tu\,\mathcal{E}} \wedge y \in C^{u\,\mathcal{E}} \,\}$
$\geqslant n S^{tu}.C^u$	$\{\, x \mid \sharp\{\, y \mid \langle x, y\rangle \in S^{tu\,\mathcal{E}},\ y \in C^{u\,\mathcal{E}} \,\} \geq n \,\}$
$\exists S^{tt}.\mathsf{Self}$	$\{\, x \mid \langle x, x\rangle \in S^{tt\,\mathcal{E}} \,\}$
$\{C^{t-1}\}$	$\{C^{t-1\mathcal{I}}\}$
R^{tu-}	$\{\, \langle y, x\rangle \mid \langle x, y\rangle \in R^{st\,\mathcal{E}} \,\}$
U^{tu}	$\Delta^{\mathcal{I}}_{t-1} \times \Delta^{\mathcal{I}}_{u-1}$
$R_1^{t_1 u_1} . \cdots . R_n^{t_n u_n}$	$R_1^{t_1 u_1\,\mathcal{E}} \circ \cdots \circ R_n^{t_n u_n\,\mathcal{E}}$

HiLog-style Satisfaction, Models

$\mathcal{I} \models \phi$ depending on type of axiom ϕ as follows:

- $\mathcal{I} \models C^t \sqsubseteq D^t$ if $C^{t\,\mathcal{E}} \subseteq D^{t\,\mathcal{E}}$
- $\mathcal{I} \models R^{tu} \sqsubseteq S^{tu}$ if $R^{tu\,\mathcal{E}} \subseteq S^{tu\,\mathcal{E}}$
- $\mathcal{I} \models \mathrm{Ref}(R^{tu})$ if $R^{tu\,\mathcal{E}}$ is a reflexive relation
- $\mathcal{I} \models \mathrm{Dis}(R^{tu}, S^{tu})$ if $R^{tu\,\mathcal{E}}$ and $S^{tu\,\mathcal{E}}$ are disjoint
- $\mathcal{I} \models A^{t-1} \colon C^t$ if $A^{t-1\,\mathcal{I}} \in C^{t\,\mathcal{E}}$
- $\mathcal{I} \models A^{t-1}, B^{u-1} \colon R^{tu}$ if $\langle A^{t-1\,\mathcal{I}}, B^{u-1\,\mathcal{I}} \rangle \in R^{tu\,\mathcal{E}}$
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- $\mathcal{I}$ is a model of a KB $\mathcal{K}$ if $\mathcal{I} \models \phi$ for all $\phi \in \mathcal{K}$

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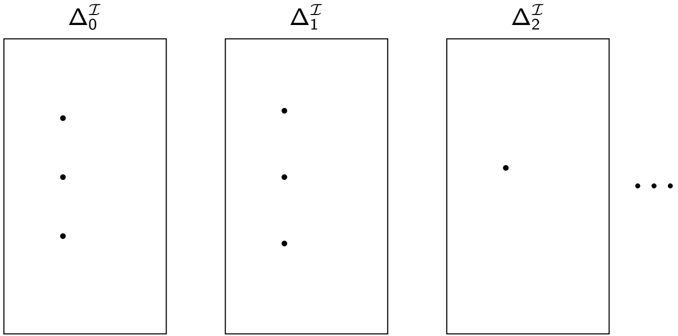
- $\mathcal{I}$ is a model of a KB $\mathcal{K}$ if $\mathcal{I} \models \phi$ for all $\phi \in \mathcal{K}$
- $\mathcal{K}$ is satisfiable if it has a model

Illustrated Intensions and Extensions in HiLog semantics



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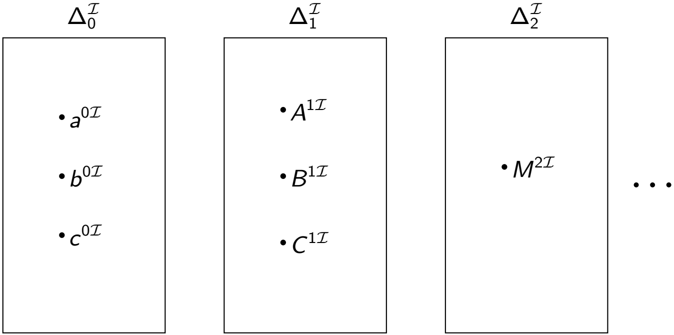

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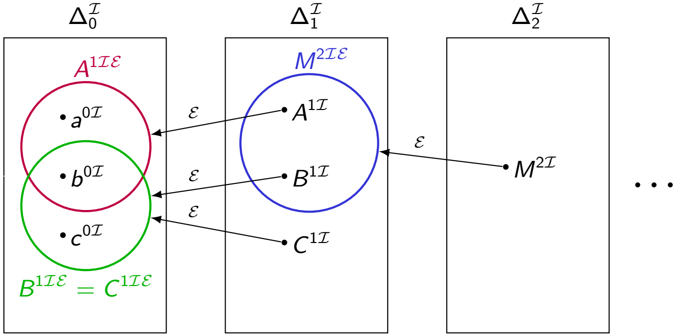

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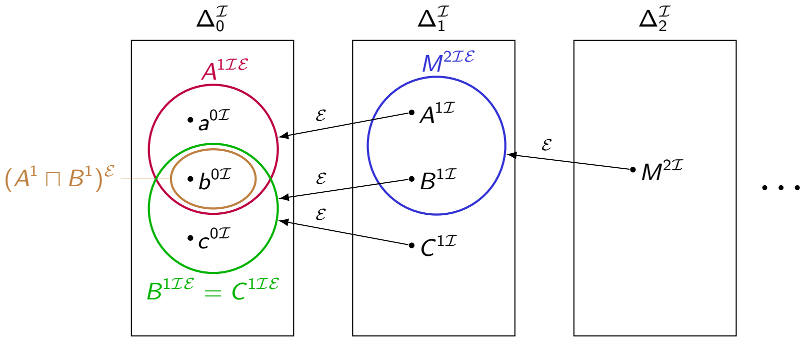

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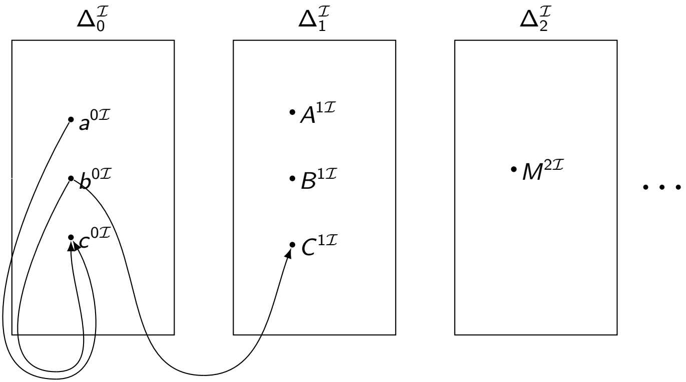

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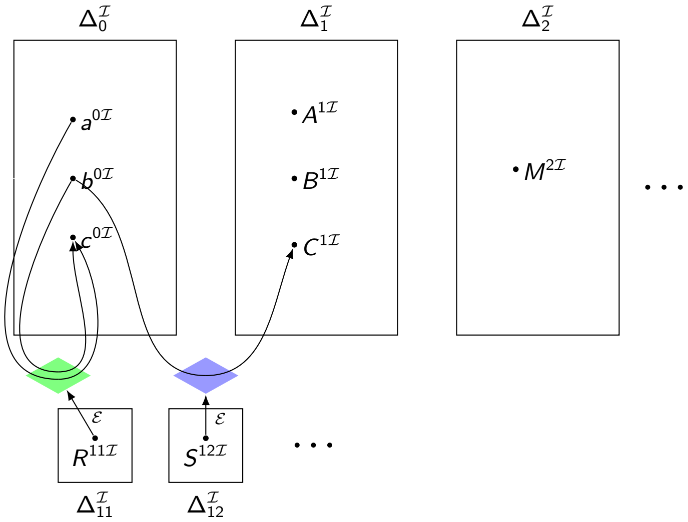

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Intension vs. Extension of a Class

So, we now have both, the intension and the extension for (atomic) classes. We also have two notions of "equality":

- intensional equality: $A^t = B^t$ iff $\{A^t\} \equiv \{B^t\}$ iff $A^{t\mathcal{I}} = B^{t\mathcal{I}}$
- extensional equivalence: $A^t \equiv B^t$ iff $A^{t\mathcal{I}\mathcal{E}} = B^{t\mathcal{I}\mathcal{E}}$

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Intensional regularity:

$$\mathcal{K} \models A^t = B^t \implies \mathcal{K} \models A^t \equiv B^t$$

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Intensional regularity:

$$\mathcal{K} \models A^t = B^t \implies \mathcal{K} \models A^t \equiv B^t$$

Extensionality:

$$\mathcal{K} \models A^t \equiv B^t \implies \mathcal{K} \models A^t = B^t$$

Intensional Regularity

Example (Motik 2007)

Consider the knowledge base $\mathcal{K}$:

$$\text{Aquila}^1 = \text{Eagle}^1$$
$$\text{Harry}^0 : \text{Eagle}^1$$
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If we see Aquila and Eagle as two different names for the same class $\mathcal{K}$ should be inconsistent

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Proposition

The HiLog-style semantics of $\mathcal{TH}(\mathcal{L})$ has the intensional regularity property.

Extensionality

Example

Consider the knowledge base $\mathcal{K}$:

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$$\mathsf{Eagle}^1 : \mathsf{Deprecated}^2$$

Most likely we don't want to derive that the other concept is deprecated as well ($\mathcal{K} \models \mathsf{Aquila}^1 : \mathsf{Deprecated}^2$ should not hold)

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Proposition

A modified HiLog-style semantics which requires injective $\cdot^{\mathcal{E}}$ has the extensionality property.

Decidability of $\mathcal{TH}(\mathcal{ALCHOIQ})$

- Assume $\mathcal{K}$ in $\mathcal{TH}(\mathcal{ALCHOIQ})$ using vocabulary

$$\Sigma = \biguplus_{0 \leq t < m} N_C^t \uplus \biguplus_{0 < t, u < m} N_R^{tu}$$

- Reduce $\mathcal{K}$ into $\Upsilon(\mathcal{K})$ in $\mathcal{ALCHOIQ}$ with meta modelling (Motik 2007) using vocabulary

$$N_a = \Sigma \uplus \{\top^t \mid 0 < t \leq m + 1\} \uplus \{\top^{tu} \mid 0 < t, u \leq m\}$$

where all $\top^t$ and $\top^{tu}$ are new concepts that will emulate the domain slices

Decidability of $\mathcal{TH}(\mathcal{ALCHOIQ})$ (cont.)

$\Upsilon(\mathcal{K}) = \mathsf{TB}(\mathcal{K}) \cup \mathsf{TC}(\mathcal{K})$

- TB($\mathcal{K}$), type-bounded version of $\mathcal{K}$, obtained from $\mathcal{K}$:
  - replace each occurrence of $\neg C^t$ in $\mathcal{K}$ with $\top^t \sqcap \neg C^t$
- TC($\mathcal{K}$) is set of typing constraints:
  - Domain disjointness:
    - $\top^t \sqsubseteq \neg\top^u$ if $t \neq u$
    - $\top^{tu} \sqsubseteq \neg\top^{vw}$ if $(t, u) \neq (v, w)$
    - $\top^{tu} \sqsubseteq \neg\top^{v}$
  - Intension typing:
    - $A^{t-1} : \top^t$
    - $R^{tu} : \top^{tu}$
  - Extension typing:
    - $A^t \sqsubseteq \top^t$
    - $\exists R^{tu}.\top \sqsubseteq \top^t$ and $\top \sqsubseteq \forall R^{tu}.\top^u$

for all $0 < t, u, v, w \leq m$, and for all A^{t-1}, A^t, R^{tu} in $\mathcal{K}$

Decidability of $\mathcal{TH}(\mathcal{ALCHOIQ})$ (cont.)

Proposition

Let $\mathcal{K}$ be a $\mathcal{TH}(\mathcal{ALCHOIQ})$ KB. Then $\mathcal{K}$ is satisfiable in the HiLog-style semantics iff $\Upsilon(\mathcal{K})$ is ν-satisfiable.

Corollary

Satisfiability in $\mathcal{TH}(\mathcal{ALCHOIQ})$ in the HiLog-style semantics is decidable in non-deterministic exponential time.

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- Typed higher-order DLs
  - Strictly separated hierarchy of types
  - Inter-type roles with homogeneous domains and ranges
- Decidability of $\mathcal{TH}(\mathcal{ALCHOIQ})$, relation to non-typed higher-order DLs
- Useful for:
  - Expressing background models of LD vocabularies
  - Other meta-modelling applications (hopefully)